CHAPTER 2
VECTORS

ROLE OF UNITS IN PROBLEM SOLVING

Units are very important in the study of physics in that all physical quantities have units. These units are classified as either fundamental units of length (m), mass (kg) and time (s) or derived units such as the joule (kg m^2/s^2).

When used in algebraic expressions, the units which accompany the numbers can be used to check not only the accuracy of the calculation but also the validity of the equation. Units will always be displayed along with the number in this review book. Remember, if the units do not work out, the solution is not correct.

SKILL M 1.1

EXAMPLE 2.1 – WHEN MANIPULATING EQUATIONS TO SOLVE FOR UNKNOWNS, USE DIMENSIONAL ANALYSIS.

Convert 10 mi/h to m/s.

Conversion factors: **1 mi = 5280 ft** **1 ft = 0.305 m**
1 km = 1000 m **1 h = 3600 s**

Each of the equalities above can be used to form a fraction or conversion factor that is equal to unity (or one). When we multiply by one we do not change the value of the physical quantities; we just express the same quantity in a different set of units. One side of the equality will appear in the numerator and the other side will appear in the denominator of a fraction. The specific way that the equality is written is decided so that the unwanted units cancel and the desired units appear in the final answer.

$$10.0\ \frac{mi}{h} = 10.0\ \frac{\cancel{mi}}{\cancel{h}}\ \left(\frac{5280\cancel{ft}}{1\cancel{mi}}\right)\left(\frac{0.305m}{1\cancel{ft}}\right)\left(\frac{1\cancel{h}}{3600s}\right) = 4.47\ \frac{m}{s}$$

The first conversion factor changes the miles to feet, the second factor changes the feet to meters, and the third conversion factor changes hours

to seconds. Notice that in each conversion factor, the numerator is equal to the denominator, and the conversion factor is equal to one.

☆ TRY IT

1 Convert 10.0 m/s to miles per hour using the factor label method as shown above.

Note: The Physics Regents Exam will not ask you to convert between the English engineering system and the MKS system. This example was included to give you a real world example of speed.

SCALARS AND VECTORS

A **scalar** quantity is a physical quantity which has **magnitude** (size) only. It is completely described by a single number plus an appropriate unit. Scalar calculations involve only ordinary arithmetic operations.

A **vector** quantity is a physical quantity that has both magnitude and direction. Calculations involving vectors require vector mathematical methods.

VECTORS	SCALARS
Displacement	Distance
Velocity	Speed
Acceleration	Energy
Force	Time
Weight	Power
Momentum	Mass
Torque	Charge

PROPERTIES OF VECTORS

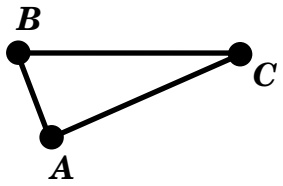
- A vector can be moved anywhere in the plane that contains it as long as the magnitude and the direction of the vector are not changed.
- Two vectors are identical (equal) if they have the same magnitude and direction.
- Vectors are concurrent if they act at the same point.
- A vector multiplied by a positive scalar quantity gives the vector with a different magnitude but the same direction.
- A vector multiplied by a negative scalar quantity gives the vector with a different magnitude and an exactly opposite direction.
- Two or more vectors can be added together to give a **resultant**. The resultant is a single vector that can replace the other vectors acting on the body and produce the same effect as the set of vectors.

- The maximum value two vectors can have occurs when the angle between the two vectors is zero degrees.
- The minimum value two vectors can have occurs when the angle between the two vectors is 180 degrees.
- The **equilibrant** is a vector exactly equal in magnitude to the resultant but in the opposite direction. When a system is in equilibrium, there are no unbalanced forces working on the system. The system may be at rest or in motion with constant velocity.

Distance and Displacement

Distance is a scalar quantity that represents the length of a path from one point to another. **Displacement** is a vector quantity that represents the length and direction of a straight line path from one point to another. Total displacement is a vector sum. A jogger is concerned with distance and a pilot with displacement. The **SI unit** for distance or displacement is the meter.

To illustrate the difference between displacement and distance, consider that an object moves from ***A*** to ***C*** along the path ***ABC***. The magnitude of the displacement is the length of the vector ***AC***. The distance the object actually moves along path ***ABC*** is greater than the magnitude of the displacement.



Solving Vector Problems Graphically

You will need a centimeter ruler and a protractor to begin. To solve vector problems graphically, first select an appropriate scale. (For example, 1 cm = 10 m.) Draw one vector with the proper angle orientation and scaled length.

Vector addition is accomplished by moving the vectors so that the tail of each successive vector in the addition is connected to the head of the next vector in the addition. The resultant vector is then drawn from the starting point of the first vector to the ending point of the last vector.

The order in which the vectors are added does not matter. All combination will give a resultant with the same magnitude and direction.

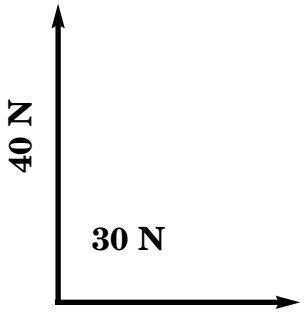
Important Definitions

Velocity is the time rate of change of displacement.
Speed is the time rate of change of distance.
Acceleration is the time rate of change of velocity.

SKILL 5.1IV

EXAMPLE – ADDING TWO CONCURRENT VECTORS

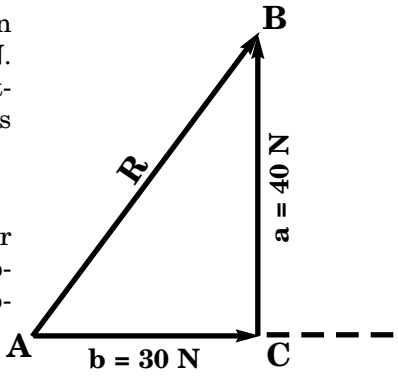
Consider two forces acting on a point. These forces are said to be **concurrent** vectors. One force acts due east and has a magnitude of 30 N. A second force has a magnitude of 40 N and acts due North. Calculate the resultant force. *Note:* before beginning any problem, it is helpful to sketch the situation. After the sketch is completed, the problem may be solved either by graphical or mathematical methods.



GRAPHICAL METHOD

Construct a scaled vector diagram. In this diagram, 1.0 cm represents 10.0 N. Add the vectors "head to tail." The resultant (**R**) starts where the first vector begins and ends where the last vector ends.

Measure the resultant with a ruler and determine the amount of force it represents. Determine the angle with a protractor.



☆ TRY IT

2 Using the graphical method, add the following vectors and determine the resultant.

a 3.0 meters north, 4.0 meters west

b 8.0 meters south, 6.0 meters east

c 16.0 meters east, 8.0 meters north

d 250. meters south, 300. meters north

e 660. meters east, 1000. meters south

f 380 cm west, 790 cm south

SKILLS 5.1, M 1.1

Mathematical Technique — Applying the right triangle rule of the preceding figure:

$$\mathbf{c^2 = a^2 + b^2}$$

Substituting:

$$c^2 = (40.\ N)^2 + (30.\ N)^2$$
$$= 1600\ N^2 + 900\ N^2$$
$$= 2500\ N^2$$

$$c^2 = 50.\ N$$

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{40.N}{50.N} = .80$$

$$A = 53°$$

The final answer is a 50 N force at an angle of 53°.

☆ TRY IT

3 Check your work for the previous graphing skill by solving (2 *a* – *f*) above algebraically (mathematical technique).

SKILL 5.1IV

EXAMPLE 2.3 – ADDING THREE VECTORS GRAPHICALLY

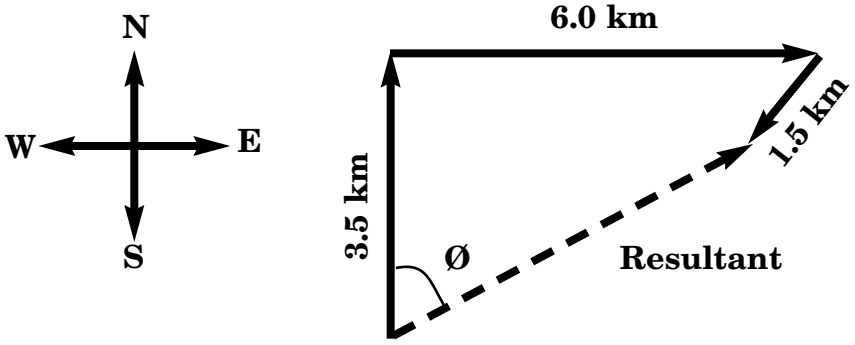
A bus heads 3.5 km north, then 6.0 km east, then 1.5 km southwest.

Draw a vector diagram to represent the bus trip. Determine the bus' displacement and distance traveled. Southwest is exactly forty five degrees south of west. This technique requires a ruler and a protractor.

Step One: Select an appropriate scale. For this example, 1 cm = 1 km was selected. The scale should allow the diagram to fit into the space provided.

Step Two: Draw each vector the correct length and the correct direction. Label each vector in the original units.

Step Three: Draw the resultant displacement vector from the original starting position to the end of the last vector arrow. Be careful to measure the length. Record this length and use the scale to convert it back to km. Measure the angle of the resultant with a protractor. Angles must be properly described to avoid confusion.



Ø = 63° East of North

Resultant = 5.5 cm = 5.5 km

Displacement = 5.5 km at 63° E of N

Note: Do not say 63° NE because the angle is not exactly 45 degrees.

Distance = 3.5 km + 6.0 km + 1.5 km = 11.0 km

☆ Try It

4 Add the following vectors and determine the resultant.

a 6 meters south, 3 meters north, 4 meters west

b 14 meters west, 8 meters south, 6 meters east

c 16 meters east, 8 meters north, 10 meters west

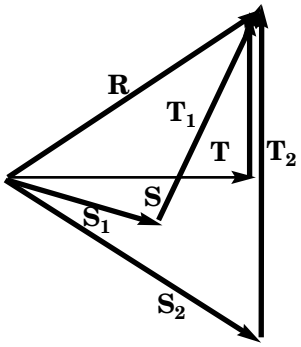
d 250 meters south, 150 meters west, 300 meters north

e 660 meters east, 100 meters south, 450 meters west

SKILL 5.1VI

EXAMPLE – COMPONENTS OF A VECTOR

Every vector can be resolved into any number of components. Most often, two perpendicular components are selected. The components represent the projection of the vector onto the **x-** and **y-axis** of a coordinate system.



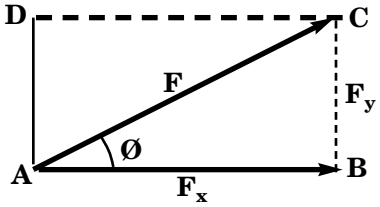
Components of a Vector:

$$\mathbf{R = S + T}$$

$$\mathbf{R = S_1 + T_1}$$

$$\mathbf{R = S_2 + T_2}$$

Consider a vector, **F**, resolved into two perpendicular vectors of magnitude $\mathbf{F_y}$ and $\mathbf{F_x}$.



Resolving a vector into two perpendicular components from trigonometry:

$$\mathbf{F_x = F \cos \text{ø}} \qquad \mathbf{F_y = F \sin \text{ø}}$$

REAL WORLD CONNECTIONS – SKILLS M1.1 AND M3.1

THE PLANE TRUTH ABOUT VELOCITY

The Concord ©PhotoDisc

A plane is traveling at a velocity of 300 m/s in a direction 30° North of East. At what velocity is the plane traveling to the north? To the east?

This problem can be solved by the graphical method or the mathematical method. Graphically, a vector whose length is 3 cm will represent the velocity of 300 m/s. The top of the page is normally considered North. First draw the vector to scale at the proper angle. Next resolve the vector into components by constructing a perpendicular to each axis. Measure the lengths of the projection along the east and north axis. This length represents the magnitude of the velocity in each direction.

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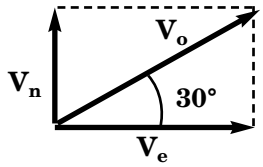
Graphical Method:



scale: 1 cm = 100 m/s

V_{north} = 1.5 cm
= 150 m/s

V_{east} = 2.6 cm

= $2.60 \text{ cm} \times \frac{100 \text{ m/s}}{\text{cm}}$

= 260 m/s

Mathematical Solution:

$$\sin 30° = \frac{V_{north}}{V_0}$$

$$V_n = V_0 \sin 30°$$

$$V_n = (300 \text{ m/s})(0.5) = 150 \text{ m/s}$$

$$\cos 30° = \frac{V_{east}}{V_0}$$

$$Ve = V_0 \cos 30°$$

$$Ve = (300 \text{ m/s})(0.866)$$

$$= 260 \text{ m/s}$$

Real World Connections – ☆ Try It

A rocket can be launched at various angles. Assume the horizontal is at zero degrees and a vertical launch is straight up. Complete the table below by determining the components of the velocity in an *x* and *y* direction for each launch angle. You may solve this either graphically or mathematically.

5 Complete the following table:

Velocity (m/s)	**Angle** (degrees)	***x*-component** (m/s)	***y*-component** (m/s)
100	10	_____	_____
100	30	_____	_____
100	45	_____	_____
100	60	_____	_____
100	90	_____	_____

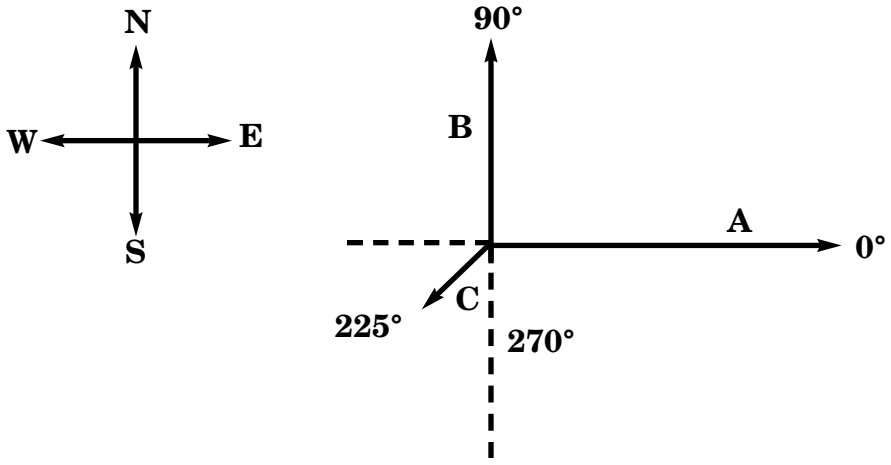
VECTOR ADDITION BY COMPONENTS

The resultant vector can also be determined by summing of the vector components. This technique will be used in the next example to illustrate how to determine the resultant of multiple vectors mathematically.

SKILLS 5.1VI, M 3.1 – EXAMPLE 2.5 – FINDING THE RESULTANT OF TWO OR MORE VECTORS MATHEMATICALLY

Note: This technique will require a scientific calculator.

A bus heads 3.50 km north, then 6.00 km east, then 1.50 km southwest. First, decide on the zero degree orientation. Determine all angles relative to that position. The zero position is typically taken as due east, but it can be any direction you choose. The reference of southwest refers to exactly 45 degrees between south and west.



Next, determine the x and y components of each vector. Set up a table like the one below and complete the entries. For this problem, the zero position was selected as due east.

Vector	Magnitude (km)	Direction (degrees)	F_x = Fcos (Ø) x-Component (km)	F_y = Fsin (Ø) y-Component (km)
Vector *A*	6.00	0.00	6.00	0
Vector *B*	3.50	90.0	0	3.50
Vector C	1.50	225.	-1.06	-1.06
		Totals	$\Sigma F_x = 4.94$	$\Sigma F_y = 2.44$

The symbol Σ (sigma) represents "the sum of."

Resultant Magnitude:

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$R = \sqrt{(4.94 \text{ km})^2 + (2.44 \text{ km})^2}$$

$$R = 5.5 \text{ km}$$

Direction is given by:

$$\tan \text{Ø} = \frac{\Sigma F_y}{\Sigma F_x} = \frac{4.94 \text{ km}}{2.44 \text{ km}}$$

$$\text{Ø} = 26.3° \text{ as measured from the due East position.}$$

This mathematical technique can be used for any number of vectors.

END SKILL

☆ TRY IT

6 A soccer player runs 53 m due south, 14 m due west, 35 m southeast and 22 m northeast. Determine the soccer player's displacement both graphically and mathematically.

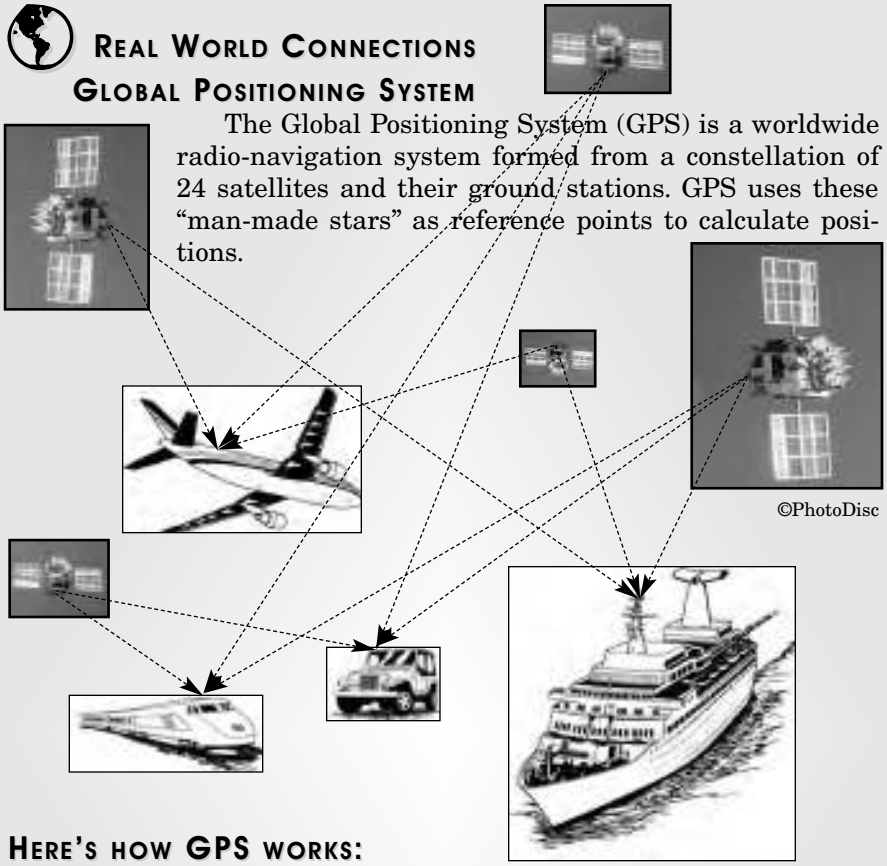
7 Determine the resultant mathematically (or graphically).
a 150 m at 10 degrees, 75 m at 45 degrees, 200 m at 80 degrees

b 500 m at 40 degrees, 280 m at 110 degrees, 375 m at 210 degrees

8 Why do you need to receive the signal from three satellites in order to determine your location?

REAL WORLD CONNECTIONS
GLOBAL POSITIONING SYSTEM

The Global Positioning System (GPS) is a worldwide radio-navigation system formed from a constellation of 24 satellites and their ground stations. GPS uses these "man-made stars" as reference points to calculate positions.



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HERE'S HOW GPS WORKS:

a The basis of GPS is "triangulation" from satellites. The signals from three satellites identify your position on Earth. A fourth satellite signal gives altitude. The intersection of these vectors can locate your position to within 1 cm.

b To "triangulate," a GPS receiver measures distance using the travel time of radio signals.

c To measure travel time, GPS needs accurate timing from atomic clocks that are precise to within a billionth of a second.

d Along with distance, you need to know exactly where the satellites are in space.

e Finally you must correct for any delays the signal experiences as it travels through the atmosphere.

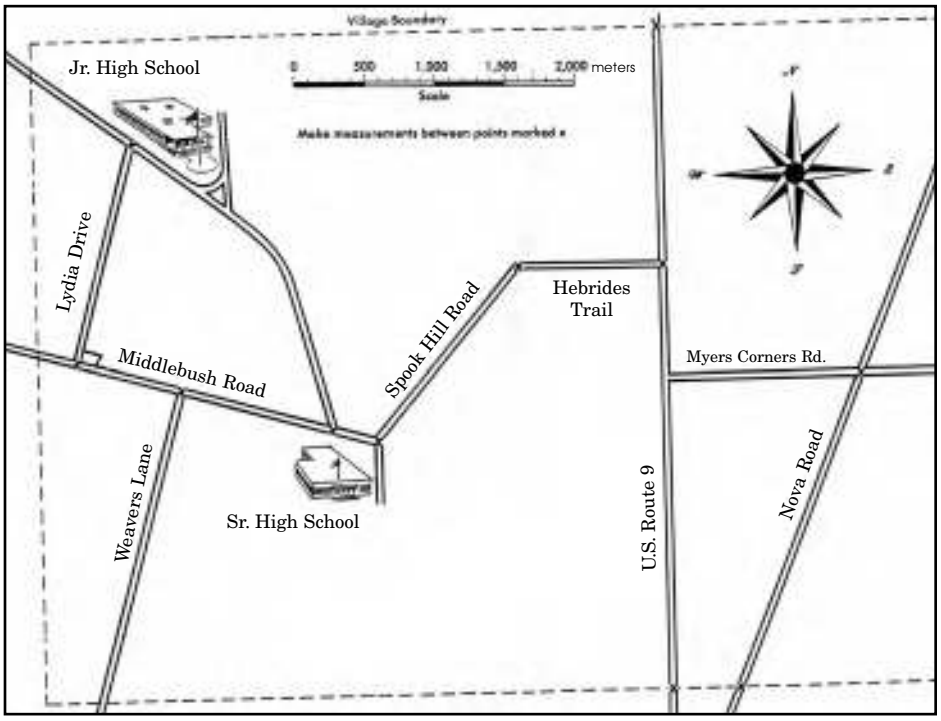
Although originally developed for the military, GPS is used for most all guidance systems and navigation, from hikers to commercial airlines and fishermen. It tracks emergency vehicles, construction equipment, and computers. It might even be in your next car.

SKILLS 5.1B,C, 5.1VI, M 1.1
LAB 3 – DISPLACEMENT VECTORS

Vectors are quantities that have both magnitude and direction. The displacement from one location to another is a vector. Vectors can be added graphically by drawing a diagram in which each vector is represented by an arrow pointing in the correct direction and of a length proportional to the magnitude of the vector. These arrows are drawn to scale and placed consecutively end-to-tip. The sum, or resultant, is represented by the vector arrow that joins the end of the first arrow to the tip of the last.

THE PROBLEM

In this lab, we shall determine the displacements between various points on a map. By treating these displacements as vectors and drawing a scale vector diagram, we can add the displacements. Then go back to the map to check the precision of your work. Carefully drawn vector diagrams can be made with good accuracy. A 1% error is a reasonable expectation. Use a sharp pencil and make measurements with care.



Add Two Vectors

Imagine that you are going from the Senior High School to the Junior High School by Middlebush Road and Lydia Drive. Find the magnitude in meters of each of the necessary displacements. Using a scale of 1.0 cm = 200 m, draw on your report sheet a vector diagram representing this trip. On your diagram, measure the resultant displacement and express it in meters. Knowing that your figure is a right triangle, calculate the result algebraically. How many significant figures should your answer have? Now return to the map and measure the actual displacement, using the map scale. Compare your results and evaluate your accuracy.

Add Three Vectors

Using a scale of 1.0 cm = 200 m, draw a vector diagram representing a trip: starting at the intersection of Weaver's Lane and Middlebush Road, travel along Middlebush Road to Spook Hill Road, up Spook Hill Road to Hebrides Trail and along Hebrides Trail to the intersection with U.S. Route 9. Compare the resultant displacement from your diagram with that obtained by making a measurement on the map.

Find the Rectangular Components of a Vector

It is often useful to represent a single vector by two others, which when combined produce the same effect. These are called the components of the original vector. Assume Myers Corners Road runs east to west. Draw a vector diagram in your notebook to obtain the answer to this question: In a trip along Nova Road from Myers Corners Road to the south boundary of the village, how far south and how far west would you travel? This process is called resolving a vector into its rectangular components.

Going Beyond

See if you can devise a method of measuring the distance from your physics laboratory to some outside feature such as a flagpole, church steeple, or water tower without leaving the building. If the object is less than a half-mile distant, the job can be done to within about 10% using only a protractor and meter stick and drawing a vector diagram to scale.

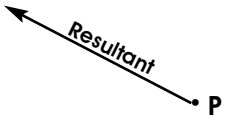
CHAPTER TWO ASSESSMENTS

PART A QUESTIONS

1 Which terms both represent scalar quantities?
(1) displacement and velocity
(2) distance and speed
(3) displacement and speed
(4) distance and velocity

2 A softball player leaves the batter's box, overruns first base by 3.0 meters, and then returns to first base. Compared to the total distance traveled by the player, the magnitude of the player's total displacement from the batter's box is
(1) smaller (2) larger (3) the same

3 The vector at the right represents the resultant of two forces acting concurrently on an object at point *P*. Which pair of vectors best represents two concurrent forces that combine to produce this resultant force vector?



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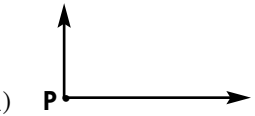

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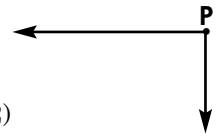

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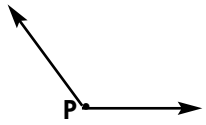

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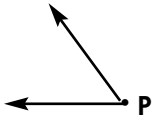
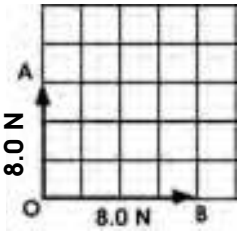


4 Two forces (*OA* and *OB*) act simultaneously at point *O* as shown on the diagram to the right. The magnitude of the resultant force is closest to
(1) 8.0 N
(2) 11 N
(3) 15 N
(4) 16 N



5 A force of 3 newtons and a force of 5 newtons act concurrently to produce a resultant of 8 newtons. The angle between the forces must be
(1) 0° (2) 60° (3) 90° (4) 180°

6 Two concurrent forces have a maximum resultant of 45 newtons and a minimum resultant of 5.0 newtons. What is the magnitude of each of these forces?
(1) 0.0 N and 45 N
(2) 5.0 N and 9.0 N
(3) 20. N and 25 N
(4) 0.0 N and 50. N

7 As the angle between a force and level ground decreases from 60° to 30°, the vertical component of the force
(1) decreases (2) increases (3) remains the same

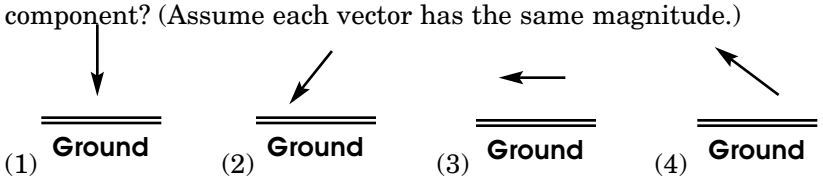
8 Which two terms represent a vector quantity and the scalar quantity of the vector's magnitude, respectively?
(1) acceleration and velocity
(2) weight and force
(3) speed and time
(4) displacement and distance

9 Which pair of terms are vector quantities?
(1) force and mass
(2) distance and displacement
(3) momentum and acceleration
(4) speed and velocity

10 Distance is to displacement as
(1) force is to weight
(2) speed is to velocity
(3) velocity is to acceleration
(4) impulse is to momentum

11 A ship changes direction several times and finishes 20 miles north of its starting point. This displacement is a vector quantity because it has
(1) both magnitude and direction
(2) magnitude but no direction
(3) direction but no magnitude
(4) neither magnitude nor direction

12 Which diagram represents the vector with the largest downward component? (Assume each vector has the same magnitude.)

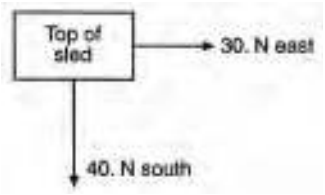


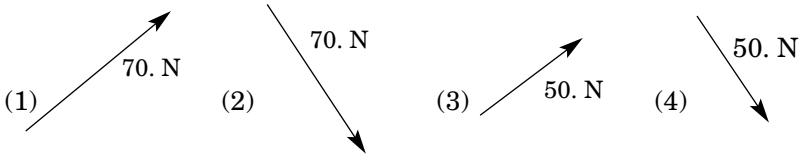
13 The maximum number of components that a single force may be resolved into is
(1) one (2) two (3) three (4) unlimited

14 A lawn mower is pushed with a constant force of F, as shown in the diagram at the right. As angle Ø between the lawn mower handle and the horizontal increases, the horizontal component of F
(1) decreases (2) increases (3) remains the same

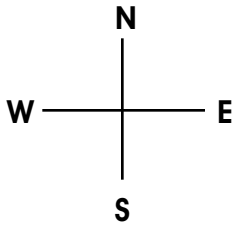
PART B QUESTIONS

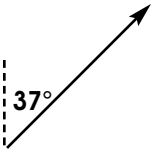
15 Two students push on a sled. One pushes with a force of 30. newtons east and the other exerts a force of 40. newtons south, as shown in the top view diagram at the right. Which vector best represents the resultant of these two forces?

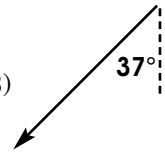


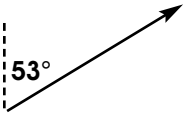


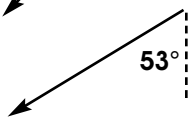
16 An object is displaced 3 meters to the west and then 4 meters to the south. Which vector shown below best represents the resultant displacement of the block?



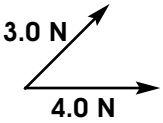
(1) 

(3) 

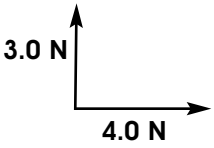
(2) 

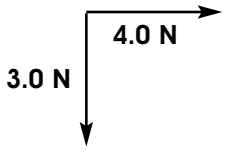
(4) 

17 A 3.0-newton force and a 4.0-newton force act concurrently on a point. In which diagram below would the orientation of these forces produce the greatest net force on the point?

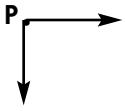
(1) 

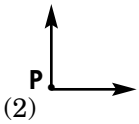
(3) 

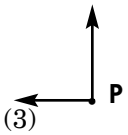
(2) 

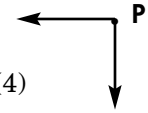
(4) 

18 The diagram at the right represents a force acting at point *P*. Which pair of concurrent forces would produce equilibrium when added to the force acting at point *P*?

(1) 

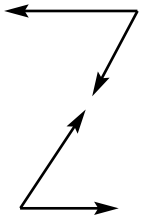
(2) 

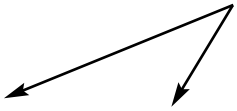
(3) 

(4) 

19 If the force vector shown in the diagram at the right is resolved into two components, these two components could best be represented by which diagram at the right?

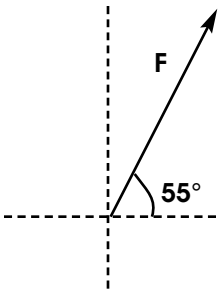
(1) 

(3) 

(2) 

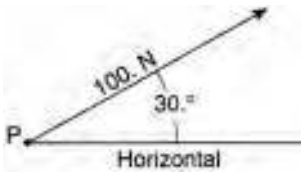
(4)

20 The horizontal component of F in the diagram at the right is
(1) F sin 55°
(2) F cos 55°
(3) F/sin 55°
(4) F/cos 55°



21 A car travels 12.0 kilometers due north and then 8.00 kilometers due west going from town A to town B. What is the magnitude of the displacement of a helicopter that flies in a straight line from town A to town B? [1]

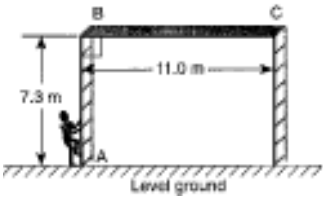
22 A 100.–newton force acts on point P, as shown in the diagram at the right.



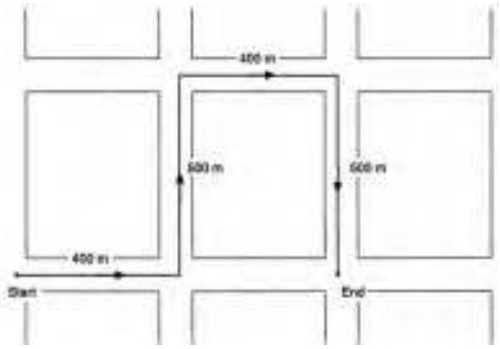
a The magnitude of the vertical component of this force is _____ N [1]

b The magnitude of the horizontal component of this force is _____ N [1]

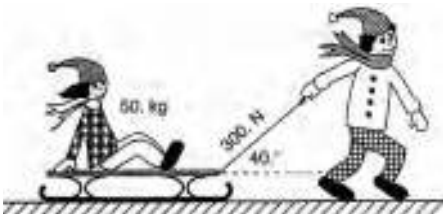
23 As shown in the diagram at the right, a painter climbs 7.3 meters up a vertical scaffold from A to B and then walks 11.0 meters from B to C along a level platform. The magnitude of the painter's total displacement while moving from A to C is _____ m [1]



24 The map at the right shows the route traveled by a school bus. What is the magnitude of the total displacement of the school bus from the start to the end of its trip?
_____ m [1]



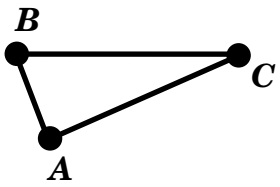
25 The diagram at the right shows a child pulling a 50.-kilogram friend on a sled by applying a 300.-newton force on the sled rope at an angle of 40.° with the horizontal.



a The vertical component of the 300.-newton force is approximately _____ N [1]

b The horizontal component of the 300 newton force is approximately _____ N [1]

26 A student follows the path ABC, as illustrated in the diagram at the right. There is a difference between the distance traveled and the displacement. What is the difference between these two quantities? [1]



_____

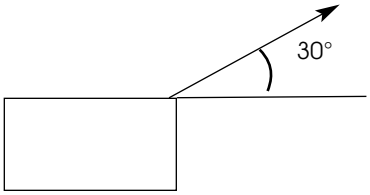
27 A student walks 3 blocks south, 4 blocks west, and 3 blocks north. What is the displacement of the student? [1] _____

28 If a woman runs 100 meters north and then 70 meters south, her total displacement will be [1] _____

29 A student walks 1.0 kilometer due east and 1.0 kilometer due south. Then she runs 2.0 kilometers due west. The magnitude of the student's distance is [1] _____

30 What is the total displacement of a student who walks 3 blocks east, 2 blocks north, 1 block west, and then 2 blocks south? [1] _____

31 A force of 100. newtons is applied to an object at an angle of 30° from the horizontal as shown in the diagram at the right. What is the magnitude of the vertical component of this force? [1]

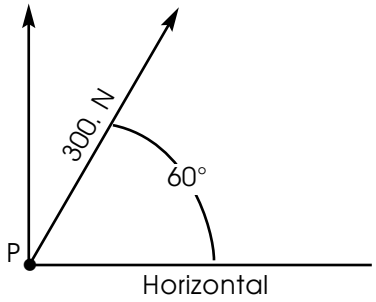


32 A plane flies 400. kilometers south and then 300. kilometers east. The magnitude of the displacement is [1] _____

33 A resultant force of 10. newtons is made up of two component forces acting at right angles to each other. If the magnitude of one of the components is 6.0 newtons, the magnitude of the other component must be [1] _____

PART C QUESTIONS

34 A 300.-newton force acts on point *P,* as shown in the diagram at the right. The magnitude of the vertical and horizontal components of this force are [2] _____ [show all work]



35 A student sailed 10. km north, 5.0 km northeast, then 6.0 km east. Select an appropriate scale and draw a graphical solution of the student's trip.

a What was the total displacement of the student? [1]

b What was the total distance the student sailed? [1]